Estimating Step Distance Using Simple Harmonic Motion

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Abstract— **Some prior studies proposed the use of Pedestrian Dead Reckoning (PDR) for indoor localization. The PDR system does not require a beacon-based infrastructure, in which a small number of sensors are put on the pedestrian. These sensors (such as G-sensor and Gyro) are used to estimate the distance and direction that the user traveled. The PDR approach can be generally categorized into two types: foot-mounted and waist-mounted. In general, the foot-mounted system can get accurate step length, but perform poorly in estimated heading direction. On the other hand, the waist-mounted system can estimate direction with high accuracy, but is hard to measure the step length. In this work, we proposed a waist-mounted based PDR to estimate step length using one 3-axis accelerometer. We utilize vertical acceleration to implement double integral for measuring the user's instant height change and use some physical features of vertical acceleration during the walking to calibrate the measurement. Based on the Pythagoras' Theorem, we can then estimate each step length based on the user's height change during his/her walking.**

I. INTRODUCTION

Indoor location sensing systems have become very popular in recent years. There were a large body of prior work in the area of indoor localization, many of them involving the use of signal strength indicator (RSSI) [1] and a large number of preinstalled devices, called beacon nodes, which periodically emit signals. While the idea of these existing systems is attractive, they require intensive pre-deployment efforts for every new building. In addition, these approaches also suffer interference and multi-path problems [2]. On the other hand, the personal dead reckoning or pedestrian dead reckoning (PDR) system is a self-contained technique for indoor localization. It is used to measure the trajectory of walker without the need to pre-install beacon nodes in the building. In other words, this technique only requires a couple of sensors to be put on the user, so that it can be used in any building without pre-installing beacon nodes or pre-building RF maps/propagation models based on surveys of the environment. Most of the PDR systems use inertial sensors (accelerometer, gyroscope, and digital compass) to measure step length and heading direction. Generally, the PDR system that uses the inertial sensors can be classified into two categories, depending on the place where the sensors are placed. For the first type, the sensors are mounted on foot. The foot-mounted method uses double integral to estimate distance and use gyroscope or compass to measure the heading direction. The second is waist-mounted, which usually detects the step event to calculate total number of steps; and then multiplies it by a constant step length which is set based on the pedestrian's characteristic (weight, height, and age) to estimate the moving distance. Some waist-mounted methods also use linear combination or linear regression to find the relationship between the acceleration, walking speed, and step length.

The PDR system has a well-know problem, called sensor drift. Due to the constraints of hardware, the inertial sensors will constantly have some small amount of errors when estimating the distance. In addition, the noise signal will further exacerbate this problem. When one use the accelerometer sensor to do double integral in estimating the distance, these errors will be accumulated. Therefore, one needs a mechanism to calibrate such errors. In foot-mounted methods, a method, called zero velocity update (ZUPT), is used to fix this problem, which is based on the characteristic of how people walk. When a person walks, one foot will stay on the ground first until the other one touches the ground. At that moment, the static one (i.e. the one on the ground) will have zero velocity in the horizontal direction. The system can use this feature to reset the calculation of velocity. However, the waist-mounted methods have no such a feature to be utilized. In other words, it will be difficult to define a moment when the sensor is static in the horizontal direction when one is walking. Therefore, the accuracy of a naïve waist-mounted method is typically worse than a foot-mounted method. On the other hand, although the foot-mounted methods are good at estimating the step length, they perform poorly in getting accurate orientation.

Existing waist-mounted methods normally estimate the distance by multiplying a fixed step length with the total number of steps. When the user is different, the system needs to be re-adjusted or re-trained. In addition, the sensor drift problem will make the results unusable for normal operation. In this paper, we propose a novel waist-mounted method that can use acceleration directly to estimate the distance and does not require any modification of the system when the user is different. Our contribution lies in using the characteristic of simple harmonic motion (SHM) to solve the sensor drift problem. We estimate the height change of the waist using the double integral of the vertical acceleration which is obtained through the accelerometer. We then estimate the horizontal walking distance from the height change of the waist using Pythagoras' Theorem. Our experiment results show that the accuracy of our distance estimation method is about 98.25%. Finally, from the practical point of view, our approach can be readily implemented on some daily devices such as PDA or smart-phone since it is waist-mounted.

This work was supported by the National Science Council, Taiwan, R.O.C. under Grant NSC 100-2219-E-006-005 and by the Ministry of Education, Taiwan, R.O.C. under the NCKU Project of Promoting Academic Excellence & Developing World Class Research Centers.

II. RELATED WORKS

Our work is built on prior work in gait recognition and pedestrian dead reckoning.

Pedometer [3] is a popular device used to monitor the number of steps during the walking. They can be mainly divided into three types. The first type uses a mechanical arm which will move up and down in response to the hip's vertical accelerations during the walking. The movement of this arm will turn on and off the circuit, so the device can detect the step. The second type is in principle similar to the first type: a plastic tube is setup in the circuit and a magnetic coil is put in the tube. When one is walking, the coil will also move up and down and it will cause electromagnetic induction to produce electric current. When the current passes the circuit, a new step is considered to have occurred. The third type uses accelerometer to detect the vertical acceleration and use some gait recognition algorithm to detect a new step event, which is similar to our work. Most recognition algorithms [4] using accelerometer are either foot-mounted or waist-mounted. They typically observe the vertical acceleration during the walking and use a threshold to detect the peak of the signal to recognize a new step event. Some pedometers can also estimate the walking distance by multiplying the total number of steps with a constant step length [3]. However, the walking pattern of each person can be different. Using a constant step length might introduce inaccuracy. In our work, we estimate the distance of each step using the simple harmonic motion and Pythagoras' Theorem.

The PDR system is used to estimate the trajectory of a person, including the walking distance, heading direction, and even the change of height. It is mainly used to improve the location accuracy in an environment where GPS is not available. Inertial sensors such as accelerometer, gyroscope, and compass, are commonly used to estimate the trajectory of a pedestrian in a PDR system. Depending on the place where the sensor is mounted on the pedestrian, we can classify the PDR system into two types: foot-mounted and waist-mounted. During the walking, when one foot is moving forward, the other one must stand on the ground to support the weight of the body. The foot-mounted methods [5] use these movement characteristics to detect the step event. To estimate the stride length, these methods typically perform a double integral on the horizontal acceleration. However, the accumulation of distance estimation [6] through double integral, called sensor drift, could introduce serious inaccuracy without further calibration. One approach that has been proposed to eliminate the cumulative error caused by sensor drift is called zero velocity update. During the walking, one foot needs to stay on the ground and remains static which the other foot swinging forward. At this moment, the angular velocity and speed of this static foot will be close to zero. Using this characteristic, when sensor detects the foot touching the ground, the system can reset the horizontal velocity to zero to calibrate the accumulated error. On the other hand, for the waist-mounted PDR system [7], one will not be able to find zero velocity in the horizontal direction, so the method used by foot-mounted PDR system can not be applied here. One common approach for waist-mounted PDR system is to use a step recognition algorithm to record the number of steps and estimate the walking distance by multiplying the number of steps with a constant step length. Since different users might have different step lengths, some prior studies [7] proposed an improvement by first collecting empirical data from different users and then use such data and linear regression to find the relation between step length, walking frequency, and the variance of acceleration. While such a method provides a higher accuracy for estimating step length, it requires the preparation of training data for every user before the PDR system can become useful. In general, the foot-mounted system can provide a higher accuracy for step length, but is not suitable to estimate the heading direction. On the other hand, the waist-mounted system can estimate the orientation of the user with a good accuracy. In addition, it can be implemented on an everyday device like smart-phone or PDA. However, it is challenging to estimate the walking distance with a waist-mounted PDR system because it can not use the horizontal acceleration directly [18]. In this work, we propose a novel waist-mounted method which does not require training for every user but the results can still be as good as the foot-mounted system.

III. METHODOLOGY

In this section, we first describe the concept of simple harmonic motion (SHM). We then discuss a pre-filtering mechanism to remove the effect of gravity on the sensor readings. Next, we describe our system architecture and algorithms in details.

A. *The concept of simple harmonic motion (SHM)*

Simple harmonic motion (SHM) [15] has been commonly used to model various motions, such as the oscillation of a spring. When an object is in simple harmonic motion, the displacement of object is proportional to the external force placed on the object and the force always points to the position of equilibrium. In other words, when the object is displaced from its equilibrium position, it experiences a net restoring force toward its equilibrium position. One characteristic of SHM is that when an object is at its largest displacement from the equilibrium position, the object's speed will become zero, and at the same time, the object will reach its maximum acceleration rate. For instance, let's put a pen on a spring and make the spring oscillate in vertical way. Then we put a long ribbon paper on the table and let the pen draw on it to show the trajectory of spring vibration. At the same time, we pull this paper in the horizontal direction at a stable speed. The trajectory shown on the paper will look like a sinusoidal wave and the vertical velocity of the highest and the lowest points of the spring will be zero. Inspired by this, if we mount a sensor on the waist of a pedestrian during his walking [16], the trajectory of sensor can be approximated by a sinusoidal wave.

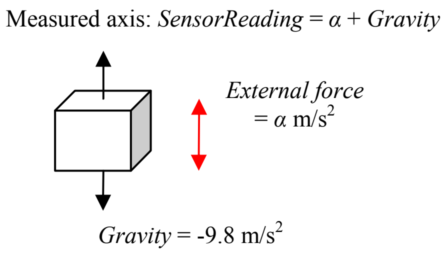

Figure 1. Gravity Elimination.

In addition, given that the velocity of the highest and lowest points of the sensor will be zero, we can utilize these characteristics to detect when the user starts a new step. Finally, once we find the points of zero velocity in the vertical direction, we can use these to do the double integral for calculating the height change of the user's waist. The height change can then be used to estimate step distance based on Pythagoras' Theorem, which will be described next.

B. Remove the effect of the gravity

During the walking, a person's body will move up and down, as shown in Fig. 4. In other words, if we assume the length of leg is *L*, the waist line will move up-and-down between *L* and *(L-h)* from the ground, where *h* is the change of the height of the waist. Considering the triangle shown in Fig. 4, formed by two feet of a person and his step length *D*. Given that *L* is known, using Pythagoras' Theorem, we can estimate *D* if we know the height of this triangle, i.e. *(L-h)*. To obtain *(L-h)*, we need to first calculate *h* which is the height change of the waist during the walking. Therefore, if we mount an accelerometer on the user's waist, the readings of the accelerometer can be used to estimate the height change h which can then be used to calculate the step length *D* based on Pythagoras' Theorem. However, there is one problem with this approach. The readings from the accelerometer will be affected by the gravity. The accelerometer is typically built on a silicon wafer in circuit to detect the acceleration. There is a poly-silicon spring on the wafer surface to provide a force against the external force. Any external force could cause the deflection of the spring. According to different levels and direction of the deflection, the values of resistance or capacitor in the circuit will change proportionally, and then the device will output the corresponding voltages which are the readings that we can measure from the sensor. However, even in the static state, the sensor can still detect the acceleration due to that the gravity constantly gives the wafer a downward force which is always 1g on earth, except when the axis (we use a three-axis accelerometer) from which we collect the reading is perpendicular the to gravity force. Therefore, to obtain the vertical acceleration of the waist during the walking for estimating height change, we need to first remove the effect of the gravity from the sensor readings. When the sensor is moving, there might be an angle between the direction of measured axis and the gravity. This angle can be from 0° to 180°. For the sake of discussion, we consider two cases: when the axis is parallel to the gravity force and when it is not.

If parallel, this means that the angle between the measured axis and the direction of gravity is 0° or 180°, as shown in Fig. 1. In this case, we can estimate the vertical acceleration caused by the up-and-down movement of the waist during the walking based on the (1). Here SensorReading is the reading we collect from the sensor for one particular axis while α is the external force that causes sensor to move up or down.

$$\alpha = Sensor\,\mathrm{Re}\,ading - Gravity(= -9.8m/s^2) \quad (1)$$

Note that since the gravity is a downward force, according to the law of inertia, when the sensor moves up, the silicon wafer will move down. Therefore, the readying α from the sensor will be more than 1g (-9.8m/s^2).

If not parallel, the angle between the measured axis and gravity is neither 0° nor 180°, as shown in Fig. 2.1, and the measurement collected from the sensor will be contributed by the components of two different forces; one is from the gravity and the other one is from the external force, as shown in Fig. 2.2. Therefore, to estimate the external force, we need to first compute the component of the external force on the measured axis, which can be achieved by subtracting the component of the gravity from the sensor reading.

$$M = gravity \times \cos\theta \quad (2.1)$$

$$N = ExternalForce \quad (2.2)$$

$$Sensor\,\mathrm{Re}\,ading = N \times \cos\theta + M \quad (2.3)$$

In the above equations, *SensorReading* is the value read from the measured axis of the sensor which is the sum of the component of *ExternalForce* and the component of the gravity. *M* is the component of the gravity which can be measured when the sensor is static.

$$N = (Sensor\,\mathrm{Re}\,ading - M) \div \cos\theta \quad (2.4)$$

$$\theta = \cos^{-1}(M / gravity) \quad (2.5)$$

We can calculate the external force based on the equation (2.4), and θ can be calculated using the Inverse Trigonometric function, as shown in the equation (2.5).

$$VerticalAcc = (Sensor\,\mathrm{Re}\,ading - M) \times \csc(\cos^{-1}(M / 9.8)) \quad (2.6)$$

Combining (2.4) and (2.5), we can obtain the vertical acceleration, as shown in (2.6). Note that here we assume the external force comes from the up-and-down movement of the waist in the vertical direction.

C. The algorithm

After the effect of gravity is removed, we can obtain the vertical acceleration generated by the up-and-down movement of the waist, which can be then used to detect each new step. Once we can distinguish every different step, we can do double integral to calculate the height change of the waist and then use that information to estimate the length of each stride based on Pythagoras' Theorem. After the walking distance is determined, combining with the orientation information from the gyro sensor, we can obtain the 2-D coordinate of the user. Before we describe our algorithm, we should first discuss how

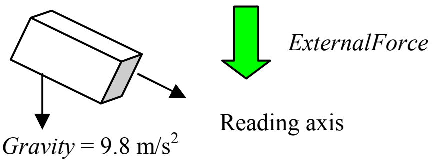

Figure 2.1. Gravity Elimination in different

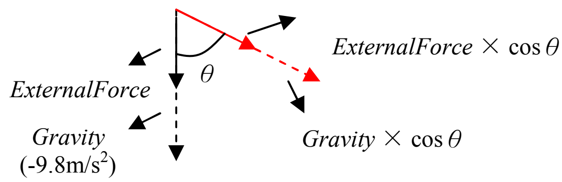

Figure 2.2. Force Diagram.

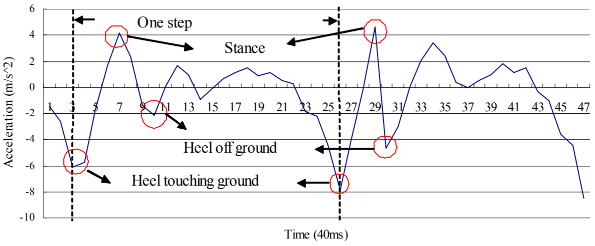

Figure 3. The vertical acceleration of walking.

we interpret the measurements of vertical accelerations during the walking. There are three major events in one step, including heel-off-ground, stance, and heel-touching-ground. As shown in Fig. 3, the lowest valley of the wave indicates when the heel is touching ground, the first peak occurs when the walker in the stance state, and the second valley of the wave indicate when the walker just moves his heel off the ground. During the walking, when a person steps forward and the heel is touching the ground, the position of the waist is in its lowest position. According to the characteristic of SHM, the vertical velocity at this position will be zero and the vertical acceleration will reach its maximum. When the body is in its stance, one foot is on the ground and the other foot is swinging forward. At this moment, the waist has the smallest displacement from the equilibrium position, so we reset the vertical velocity at this point. After that, the body starts to lean forward and the foot which was on the ground is now on its tiptoe, which will give a force to push the body up, so that the vertical acceleration will change to the opposite direction and cause another valley in Fig. 3, due to the law of inertia.

1) Stride length estimator

When a new step event is detected, the system can compute the step length using the double integral: the integral of acceleration will give us the velocity and we can get the distance of the step from the integral of the velocity. However, to avoid that the errors from the results of the double integral are accumulated, we need to do zero velocity update (ZUPT) to avoid the error accumulation. Given that the sensor mounted on waist is in a simple harmonic motion, we can find it highest (the first peak in Fig. 3.) and lowest (the first valley in Fig. 3) positions and reset the vertical velocities of these two points to be zero.

Nevertheless, since we consider the height change of the waist as a simple harmonic motion from its highest position to the same position again, if we simply do double integral, the displacement will become zero or close to zero. Therefore, we calculate the absolute value of current velocity and use it to do the next integral. In equations (3) and (4), the n is the number of acceleration sample, the V_n is the vertical velocity, a_n is the vertical acceleration, and h is the height change. We can get the velocity by doing integral of vertical acceleration. In equation (4), we use absolute value of velocity to do another integral and consider that the height change only result from ascending and descending of the body movement. The obtained result is then divided by 2 to get an average of height change. Here we assume that the height change in the directions of ascending and descending are approximately the same. Finally, we use the obtained height change to estimate stride length based on the Pythagoras' Theorem. As shown in Fig. 4, if we assuming that the known leg length of is L and the height changing is h, we can use the equation (5) to calculate the step distance D.

$$V_n = \int_1^n a_n \; dt \tag{3}$$

$$h = (\int_1^n |V_n| \; dt)/2 \tag{4}$$

$$D = 2 \times \sqrt{L^2 - (L-h)^2} \tag{5}$$

In our system, we consider each step can be divided into two parts, from stance to stride and then from stride back to stance. So if we consider that a user move from point A to point B, then the first step and the last step are actually only half a step, because the first step is only stride-to-stance and the last step is only stance-to-stride. Therefore, we consider the first step and the last step as the special cases. If the system detects that the user is currently in his first or last step, the height change will not be divided by two. In addition, there is only one point where the velocity needs to be reset in these cases. The initial velocity of first step is already zero and the last step does not need to reset the velocity when the body is in the stance state. Finally, during the walking some unpredictable body vibrations might interfere with the sensor value. We utilize a threshold-based low pass filter [17] to remove these noises.

IV. EXPERIMENTS AND RESULTS

In this section, we discuss the results of our experiments for the proposed method. We performed two experiments. For the first experiment, the user walked in a straight line. We performed two set of tests. One is less than 10 meter and the other is longer than 10 meter. In the second experiment, we combine the orientation information (from gyro) to estimate the location of the user in a 2-D space.

A. Hardware

The sensor platform we used is Taroko which is a modification version of TelosB mote [8] and designed originally by UC Berkley. Taroko is a programmable, low-power wireless sensor platform. The microcontroller unit of Taroko is TI MSP430 F1611 with 16 bit RISC [9]. MSP430 has 48K bytes flash memory and 10K bytes RAM that supports the serial communications such as UART, I2C, SPI, and Digital I/O. Besides, Taroko is equipped with the CC2420 RF transceiver [10] which is a low cost device for wireless communication in the 2.4GHz band based on IEEE 802.15.4 [11]. The maximum radio distance is around 100m. It also supports the USB interface using FTDI chip [12]. Taroko can use the USB interface to connect to a computer for powering, program upload and data collection. In this work, we use an

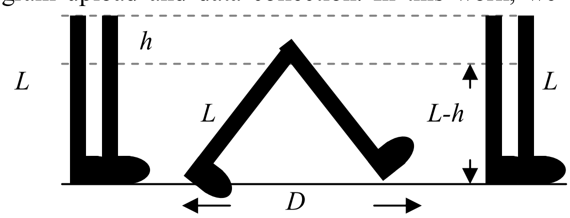

Figure 4. Walking Diagram.

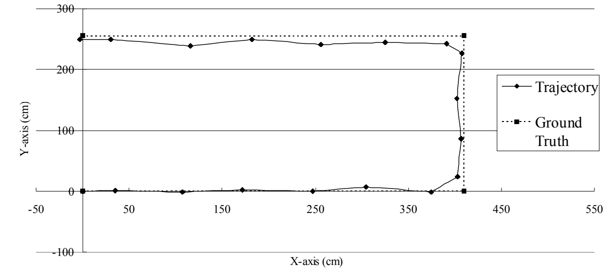

Figure 5. Trajectory.

extended board to combine the accelerometer (ADXL330 [13]) and gyroscope (IDG500 [14]). The ADXL330 is a small, low power, 3-axis accelerometer with signal conditioned voltage outputs, all on a single monolithic IC. It can measure acceleration with a minimum full-scale range of ±3g. The IDG-500 is an angular rate sensor and its full scale range is ±500°/s. Our PDR algorithm is implemented on the sensor to reduce the chance of packet loss when sending sensor info back to a computer for remote processing.

B. Straight distance measurement

In this experiment, we performed two set of tests. One is less than 10m and the other is more than 10m. The former is repeated for 20 times and the latter is repeated for 5 times. To obtain the ground truth, we used a laser distance meter to measure the distance. During each trial, the user put the sensor on his waist and walked in different speeds. The estimated distance is based on the results computed by the sensor, and the ground truth is obtained from the laser distance meter. Our results show that the average of accuracy is about 98.26% and the standard deviation is 1.09%. The average accuracy for the longer-distance case is similar to that of the short-distance case. The average of accuracy is 98.25% and the standard deviation is 1.29%. We also compare our results with the existing foot-mounted methods. Our accuracy is close to the best results from the foot-mounted method [5] (about more than 98%) and better than all the existing waist-mounted methods (for example, the accuracy in [7] was about 96%). Note that the foot-mounted method is generally not good for estimating the direction of heading, as discussed previously. Therefore, it will be difficult if one wants to employ a foot-mounted method for indoor localization.

C. Coordinate test with ground truth

In this experiment, we combine the distance measurements from the accelerometer measurements together with the direction information from gyro to calculate a person's 2-D coordinate. The experiment results are shown in Figure 5. The solid line is the trajectory and the dashed line is the ground truth. The total distance of real route is about 10.75m, and the estimated distance of trajectory is about 10.80m.

V. CONCLUSION AND FUTURE WORK

Based on the characteristic of human walking, we utilize simple harmonic motion and Pythagoras' Theorem to estimate the user's step length. The novelty in our approach is that, in addition to using the vertical acceleration to detect the step event (for counting the number of steps like a pedometer), we make use of the vertical acceleration directly to estimate the step length. As compared to a foot-mounted method, our scheme provides a better platform for estimating both the user's walking distance and orientation and is more applicable for applications of indoor localization. In this work, we do not consider the calibration of the user's direction. In our future work, we plan to use the information of building map to calibrate the gyro drift.

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